Thermal Radiation in Absolute Measure at Very Low Temperatures

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X. *Thermal Radiation in Absolute Measure at Very Low Temperatures.*

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Contents.

Introduction.

1. The experiments described in the following paper form part of an investigation with which one of the present authors has been engaged for many years past. The object of this investigation is the direct determination, in absolute measure, of the loss of energy, from a heated body to cooler surroundings, under differing conditions as to (1) the dimensions of the cooling body; (2) the nature of the surfaces of the cooling body and of the enclosure; (3) the mean absolute temperature of the cooling body and the envelope; (4) the nature of the atmosphere surrounding the cooling body. Several papers have already been published on this subject in the 'Philosophical Transactions of the Royal Society' and in the 'Proceedings,' and have been communicated to the British Association and elsewhere.* In all of these the lowest temperature available was the atmospheric temperature, or rather the temperature of the water supply of the laboratory; and the body from which radiation was to take place was heated to a temperature as much above the atmospheric temperature as the circumstances connected with the experiment would, from time to time, permit. Thus the upper and lower limits of these experiments have been something below 1000° C. as the highest temperature of the cooling body, and, say, 10° C. as the lowest temperature of the enclosure.

2. Some time ago, through the kindness of Lord Blythswood, F.R.S., an unlimited supply of liquid air was placed at our disposal, to be followed later by a supply of liquid hydrogen. It seemed therefore most desirable to extend the research in the

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* 'Phil. Trans.,' 1887 and 1893; 'Brit. Assoc.,' 1884, 1901, 1905, &c.

direction indicated by the new facilities given to us, which both greatly extended the limits governing the enquiry, and also made it possible to obtain determinations, in absolute measure, of thermal radiation between bodies at extremely low temperatures. It seemed specially interesting, for example, to determine, in absolute measure, the radiation under given circumstances from a body at, say, ordinary atmospheric temperature, to an enclosure at a temperature which may perhaps approximate to that of space. With the exception of some preliminary results, which were communicated 18 months ago to the British Association, nothing of the kind has been published, so far as we are aware.

3. The main object of the present paper is to give an account of this research. We have also, however, made an endeavour to sum up, and bring into co-relation, the results of earlier experiments, made under various circumstances, with forms of apparatus differing widely from each other, with different radiating surfaces and different enclosures, or sometimes with the same apparatus after it has been dismantled and re-erected. Finally, we have made an attempt to apply the law of STEFAN to the results, in order to obtain some idea as to how far the conditions under which it holds have been realised in these experiments.

DESCRIPTION OF THE APPARATUS EMPLOYED.

4. Although the apparatus used has already been described in former papers, a short description of it here will probably be of advantage, particularly as it was

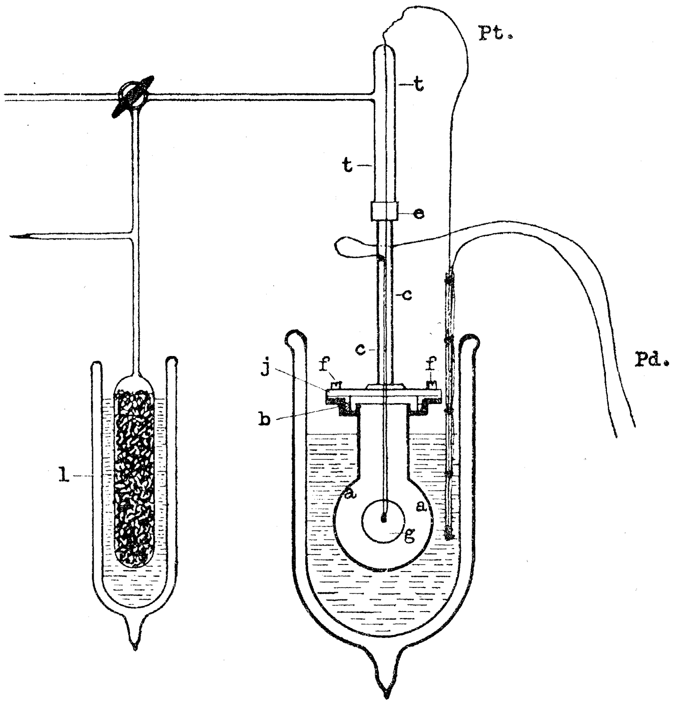

Fig. 1.

necessary to alter the arrangements considerably, in order to make them suitable for low-temperature work.

5. The bodies used in these experiments for radiation purposes are solid copper globes, 4 centims. in diameter. These globes, and the enclosure in which they are suspended, were used in many earlier experiments on radiation at high temperatures;

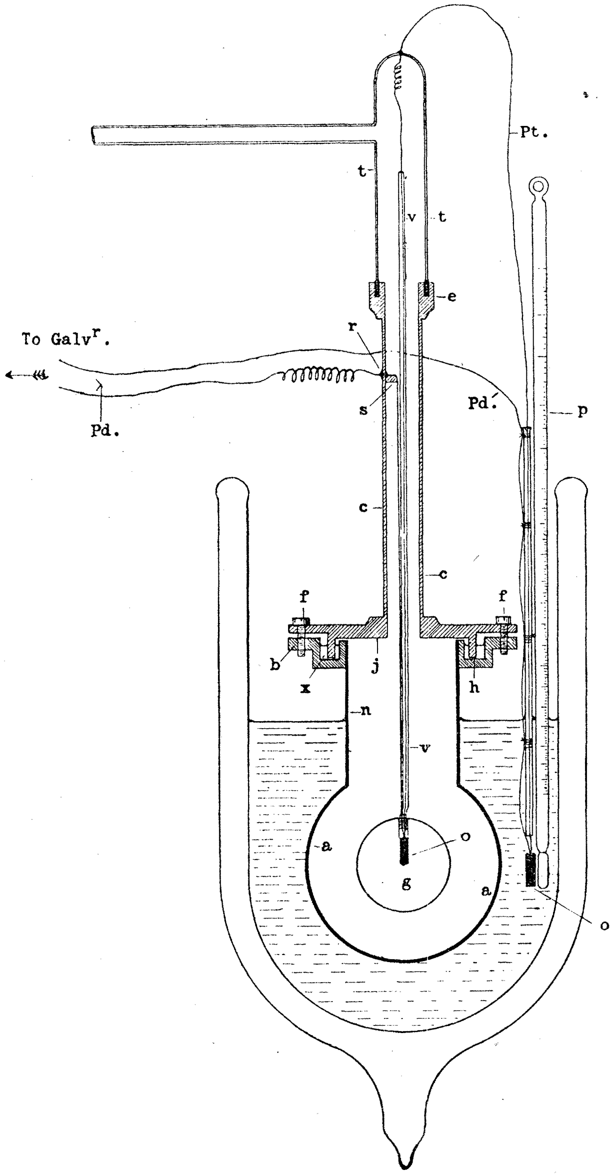

Fig. 2.

and the main parts of the apparatus used then and of the present apparatus are essentially the same. As has been already mentioned, however, modifications had to be introduced, owing to the low temperature to which the enclosure was subjected.

6. Referring to the general sketch of the apparatus, fig. 1, and to fig. 2: at the centre of a spherical copper shell *aa*, 8 centims. in diameter, a copper globe *g* is suspended. The

enclosure was beaten into shape out of a piece of thick sheet copper. It has a brass, cup-shaped flange, *b*, brazed on to the neck *n*, and the whole is thoroughly tinned both inside and outside, to make sure that every crack or pin-hole shall be filled in and the enclosure made vacuum tight. A seamless brass tube, *cc*, about $1\frac{1}{4}$ centims. in internal diameter, has the corresponding flange, *j*, brazed on to its lower end, and has a collar, *e*, with a U-groove turned in it, soldered to its upper end. When the enclosure is brought up into position, the shoulder of the upper flange, *j*, fits into the cup of the flange on the enclosure, and the two are firmly clamped together by the four set-screws *ff*. The joint is then made vacuum-tight by pouring molten fusible metal into the cup, *x*, the flanges having been previously prepared by tinning them with fusible metal.* Fusible metal was used on account of its low melting-point, as the prolonged heating necessary to make a good soldered joint of such extent has a tendency to tarnish the highly polished silvered globe when it is suspended within the enclosure. A piece of wide glass tubing, *tt*, was ground to fit the groove in the collar *e*; and, when all other preparations were complete, the joint was carefully sealed with "Siegelwachs." To prevent the "Siegelwachs" from becoming brittle by the cold conducted up the brass tube *cc*, when the enclosure is immersed in a bath of liquid air, copper vanes were clamped to the upper part of the brass tube to act as a radiator. A T-piece of quill tubing was sealed on to the glass tube *t*, to connect the enclosure to a pair of five-fall Sprengel pumps which are arranged to run continuously. Interposed between the pumps and the enclosure is a M'Leod gauge† for measuring the vacuum pressure; and attached to a three-way stopcock there is a bulb, *l*, filled with cocoanut charcoal, which can be put in communication with the enclosure.

7. All the joints between the various pieces of glass apparatus are fused together with the blow-pipe, so as to avoid rubber, or other such connections.

Thermojunctions.

8. For measuring the difference of temperatures between the globe and the enclosure, a pair of platinum to platinoid thermojunctions are used. These two metals give an excellent thermo-electric combination, as was shown in a paper on the subject by J. T. BOTTOMLEY and A. TANAKADATE.‡ We have found them very satisfactory.

The two thermojunctions are soldered into two little copper cylinders, *oo*, about

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* While working with fusible metal we have found that this material becomes springy at liquid-air temperature, and thus yields elastically to any changes in shape of the flanges.

† An improvement was made in the present apparatus by placing the M'Leod gauge as close as possible to the enclosure of which the vacuum pressure is to be measured by the gauge. We have found, from past experience, that it takes a considerable time for small differences of pressure to become equalised in highly exhausted enclosures, especially when the rarefied gases have to diffuse through a length of glass tubing.

‡ 'Roy. Soc. Proc.,' vol. 46, 1889, p. 286.

1 centim. long and 0·4 centim. in diameter. One of these cylinders is screwed into the centre of the copper globe, and the other, carrying the outside thermojunction, is tied to the bulb of a pentane thermometer p, and is placed in the bath of liquid air which surrounds the enclosure.*

9. The cooling globe g is suspended by the wires of the thermojunction. Its weight is borne by the platinoid wire, which passes through a fine hole drilled in the wall of the brass tube, and the wire is guided to hang centrally in the tube by the support s. After the length of the platinoid wire has been adjusted so that the copper globe shall hang at the centre of the spherical enclosure, the wire is soldered at the point r, where it passes through the wall of the tube. The platinum wire passes out through a sealed joint in the roof of the glass tube tt, and to insulate it from the platinoid wire, and from the brass tube in which the two hang side by side, the platinum wire is enclosed in a fine glass tube or sleeve, v, which completely protects it.

10. The ends of the thermojunction circuit are brought to a mercury reversing key, which has four cups. To the copper electrodes of one pair of cups the thermojunction wires are soldered; and the electrodes of the other pair of cups are joined up, through an Ayrton Universal shunt, and a 100,000 ohms resistance, to a standard Clark cell. Two amalgamated copper rods, the terminals of the galvanometer circuit, can be revolved so as to make contact with either pair of mercury cups. This allows the constant of the galvanometer, and the thermojunction value, to be checked with reference to the value of the standard Clark cell whilst the experiment is proceeding. No joints exist between the thermojunctions and the cups of the mercury key, the platinoid wires of the thermojunctions being taken direct to the mercury cups; and wherever there are joints in this circuit they occur in pairs, and the corresponding joints are insulated and placed side by side, and then wrapped in a thick protective covering of cotton wool. All connections exposed to draughts, or likely to give rise to thermo-electric currents, were covered up with cotton wool.

Galvanometer.

11. Owing to the passing of electric trams in the vicinity of the Laboratory, we were obliged to abandon the dead-beat Thomson reflecting galvanometer of the old pattern (having a small mirror backed with four tiny magnets, hung by means of a spider line, at the centre of the galvanometer coil), which was used on former occasions and found thoroughly satisfactory. A Kelvin marine type of moving-coil galvanometer was substituted for it, and was made extremely sensitive and dead-beat; and was also adjusted to give proportional deflections. This, used with a Steinheil telescope, placed 1 metre distant from the galvanometer mirror, and with

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* To prevent the thermojunction wires from touching one another in the liquid air bath, the platinum wire is enclosed in a piece of thermometer tubing, and the platinoid wire is tied at intervals to the outside of the tube. This allows the junction to be moved about in the bath without fear of disturbing the wires.

a circular scale 1 metre in radius ruled in half millimetre divisions, gave results which left nothing to be desired. The telescope, galvanometer, and each part of the radiation apparatus, were fastened firmly down, to prevent any alteration taking place after the thermojunctions had been calibrated.

Calibration of the Thermojunctions.

12. To calibrate the thermojunctions for low temperatures was not a simple matter. The difficulty was to get a number of reliable points between the temperatures 0° C. and −200° C. The method ultimately adopted is similar to that used, in J. T. Bottomley's earlier experiments on radiation, for determining the resistance of a platinum wire at high temperatures; but it was suitably modified for the present requirements, and for dealing with low temperatures.

13. A thick double-walled copper tube was taken, and one end immersed in a Dewar flask filled with liquid air, fig. 3. The end *a* projecting out of the flask was at atmospheric temperature, say 16° C.; and the portion immersed, *b*, at −194° C., the boiling point of liquid air. Between the points *a* and *b*, along the length of the tube above the surface of the liquid air, there was a fairly uniform temperature gradient, which was maintained with the greatest steadiness.

One of the little copper cylinders *o*, into which the thermojunctions are soldered, was tied to the bulb of a pentane thermometer *p*, reading from +30° C. to −200° C. This thermometer had previously been compared with a standardised Reichsanstalt pentane thermometer, and checked by a hydrogen thermometer, and by a platinum resistance thermometer. The other thermojunction was within the copper globe, and inside the enclosure, as shown in the diagram. In taking observations, the junction in the copper globe was kept at a known fixed temperature by surrounding the copper enclosure with ice, or water at the temperature of the laboratory, or with liquid air. The pentane thermometer with the junction attached was lowered into the copper tube, and, by placing it at different heights, series of temperatures were obtained which remained absolutely steady whilst the galvanometer readings were being taken. By raising or lowering the flask of liquid air, or by adjusting the height of the copper tube above the surface of the liquid air, or by raising or lowering the little copper cylinder from point to point in the copper tube, it was easy to find any required point of temperature throughout the whole range of the pentane thermometer. When the readings of the pentane thermometer and galvanometer showed that the thermojunction in the copper tube was at a steady temperature, a stop-watch was started; and if at the end of two minutes no alteration had taken place in the thermometer and galvanometer readings, the galvanometer deflections to the right and left of zero were observed, and then the zero was taken.*

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* This method of obtaining a series of known fixed temperatures, high or low, is most convenient; and is far superior to any other with which we are acquainted.

14. The results obtained were plotted as a curve, taking the sum of the galvanometer readings as ordinates, and the difference of temperatures between the thermojunctions as abscissæ. From this curve the temperatures corresponding to the observed galvanometer deflections were read off when it was required to convert thermojunction readings to temperatures on the Centigrade scale.

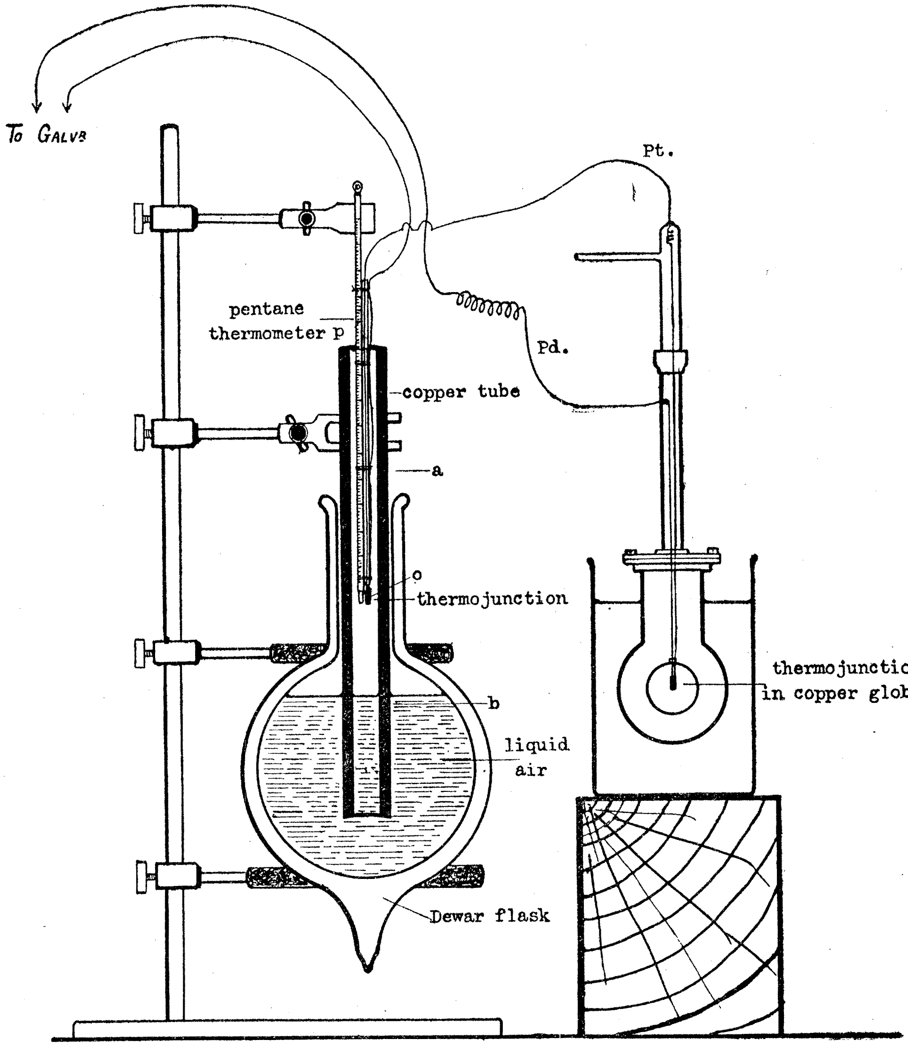

Fig. 3. Calibration of thermojunctions.

This curve was plotted on a large scale on a table covered with a sheet of plate glass, 4 feet 6 inches by 3 feet 6 inches, ruled in 10-centimetre squares. The glass plate was not wholly covered with squared paper; those squares only through which the curve passed were filled in. The smallest division of the squared paper is equal to 1 mm., and this corresponds to 0·1° C., or one scale division of the galvanometer.

15. The results of confirmatory experiments, made after intervals of months, agreed with absolute accuracy; which showed that both the thermojunctions and the galvanometer were quite constant.

METHOD OF CONDUCTING AN EXPERIMENT.

16. Before commencing an experiment, the apparatus, and the tubes connecting it to the Sprengel pumps, are thoroughly dried out. The sooted or silvered globe, as the case may be, is then attached to the copper plug *o*, into which the thermojunction is soldered. When an experiment is being made with the sooted globe, the globe receives a fine dead-black coating of soot from a small gas jet when it is hanging in position on the thermojunction wires. The enclosure *aa* is also given a coating of soot on the inside, and is then brought up into place and clamped to the upper flange by the set screws *ff*, which pass through holes drilled in lugs cast on the flanges. The joint *h* between the flanges is next made tight by pouring molten fusible metal into the cup *x* till the joint is well covered. A small blowpipe jet is run over the surface of the fusible metal just after it has set, to make sure that no pin-holes are left in the joint.

The preliminary exhaustion of the apparatus is made with a two-cylinder Fleuss mechanical pump, which very rapidly produces almost a barometric vacuum. Then the Sprengel pumps are set working, and they continue to run until a high vacuum is obtained. When, after repeated heating, the last traces of moisture and the occluded gases have been driven from the enclosure and charcoal bulb, the pumps are shut off temporarily, and the M'Leod gauge and charcoal bulb, *l*, are left in communication with the enclosure. The charcoal bulb is then immersed in a vessel of liquid air, which causes the collapsible gases to be absorbed by the charcoal and condensed, or frozen. This greatly improves the vacuum, and it clears the enclosure of traces of mercury vapour which may have diffused through from the pumps, a matter of the highest importance.

17. The next operation is to check the galvanometer with the standard Clark cells, in order to ascertain that its constant has not altered since the calibration of the thermojunctions. When this has been done, liquid air is poured into the Dewar vessel surrounding the enclosure, and as soon as the violent ebullition has subsided, denoting that the enclosure has cooled down to the temperature of the liquid air, observations commence.

18. Readings are taken every 2 minutes at the beginning of the experiment, and later on at intervals of 5 minutes, the frequency depending on the state of the vacuum, and the rate of cooling of the globe.

A chronometer, beating half-seconds, is placed close to the observer at the reading telescope. At 15 seconds before the exact minute the mercury switch is closed and the deflection to the right of zero observed. The switch is instantly reversed, and at 15 seconds after the minute the galvanometer has come to rest and the deflection to the left of zero can be read. The switch is then opened and the zero of the galvanometer is taken. The vacuum pressure, and the temperature of the liquid air, are read at intervals and noted alongside the readings with which they correspond. Also, the observations, as they are made, are plotted on squared paper. The cooling curves thus obtained afford a good check as to the working of the thermojunctions, while they keep the observer informed as to the steadiness of the cooling process.

19. To allow for evaporation of the liquid air, and to keep the level in the Dewar vessel up to a fixed mark on the enclosure, a small quantity of liquid air is added every 5 minutes. The temperature of the enclosure is kept constant by this method. It was found that adding fresh liquid air in large quantities caused irregularities on the cooling curves, which were observable as the readings were plotted.

20. Trouble was sometimes caused by the leaking of the copper shell, which, owing to the extreme temperature to which it was cooled, would get strained, and allow a slight leakage to take place at the joints or the brazed seams. These leakages disappeared when the liquid air was removed. They were, however, often overcome, and the vacuum kept up to a constant pressure by the Sprengel pumps and the charcoal bulb. Latterly the leakage was got rid of by keeping a layer of pentane over the fusible metal covering the joint between the flanges, and by painting the outside of the enclosure with collodion varnish.

Calculation of Results.

21. We will now explain the method of calculating the results. For this purpose we use an equation of the form

$$-c\frac{dv}{dt} = e\mathrm{S}\,(v-\mathrm{V}),$$

where v is the temperature of the cooling globe, V that of the enclosure, c the capacity for heat of the globe, and S its surface, while e is a coefficient which has been called the *Emissivity.* It is easy to show by an application of Fourier's equations that, to a first approximation sufficient for our present purpose, the loss of heat by conduction by the thermojunction wires may be allowed for by adding to S, or rather eS, a small quantity depending on the diameter, conductivity, and emissivity of the wires, the length of the wires being so great that no heat passes away by conduction at the ends.

22. A point requiring consideration is the following :—The arrangements explained above give approximately the temperature at or near the centres of the copper globes. In a criticism of D. Macfarlane's experiments, which were carried out in 1872 with the same globes which we have used and under circumstances precisely similar, M. Cornu, 'Journal de Physique,' December, 1873, raised an objection to the thermoelectric method as compared to the method used by Dulong and Petit, where the bulb of a thermometer was a cooling body, on the ground that the temperature at the

centre may be different from that of the surface.* This objection was completely answered by numerical calculations founded on the dimensions and conductivities of the globes in question; and, in an interesting appendix, the whole question is discussed in 'Roy. Soc. Proc.,' June 10, 1875.†

23. In the above formula the coefficient e corresponds to the *emissivity*; it is the quantity of heat lost per second, per square centimetre of surface, per degree centigrade of difference of temperatures of radiating surface and surroundings. This formula is commonly taken to be a representation of the "law of cooling," whether in air or in any other gas, or in vacuum, the range of temperature dealt with being moderate. The numerical value of e depends on the circumstances under which the cooling takes place, and, when air is present, on the dimensions and shape of the cooling body.

24. One way of dealing with the matter, from an experimental point of view, is to solve the equation above as though e were constant—which is, of course, approximately true if the difference of temperatures is small—and then to determine numerically the value of e at different places in the temperature scale, taking an exact account of the circumstances. When a sufficiently large number of such values have been obtained, a basis for a wider theory will have been laid.

Taking, then, the equation

$$-c\frac{dv}{dt} = eS(v-V),$$

and its solution

$$\log_\epsilon \frac{v_0-V}{v-V} = \frac{eS}{c}t,$$

where v_0 is the temperature of the cooling globe when $t = 0$, and V the temperature of the enclosure, we have for the numerical calculation of e, using common logarithms, M being their modulus,

$$e = \frac{Mc}{St_1}\{\log(v_0-V) - \log(v-V)\}.$$

Here $t_1 = 300$, the interval of time used in our experiments being 5 minutes; $c = 25{\cdot}482$, and $S = 50{\cdot}26$, to which we must add a small quantity which we roughly estimate at about 0·6 per cent.—the correction applied for the carrying away of heat by the thermojunction wires. This we calculate, assuming that the emissivity of the surface of the conducting wires is much the same as that of a tolerably clean silvered surface.

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* To find experimentally how long it would take for a wave of heat to travel from the surface to the centre of the copper globe, a spirit flame was applied to the outside of the globe, and the instant of application was taken with a stop-watch. The flame was kept under the globe for 2 seconds, and in 6 seconds the zero of the galvanometer scale was seen to be moving rapidly across the field of the observing telescope. The maximum deflection was obtained in less than 20 seconds from the time of applying the spirit flame.

† Republished in Lord KELVIN'S 'Collected Papers,' vol. 3, p. 245.

25. The emissivities in the present paper are calculated by two methods. To explain the working out of the results we cannot do better than quote two specimen pages out of the calculation note-books. These pages are not selected because they show any special feature; they are simply average specimens.

Method 1.

26. This method of calculating the emissivities is the one chiefly used in the former paper, 'Phil. Trans.,' 1893. It is fully explained and illustrated there in an appendix, and a brief description of it here will suffice. Referring to p. 360: Column 1 contains the times, against which the corresponding galvanometer deflections are noted in Column 2. These deflections correspond to the differences of temperatures between the thermojunctions,* and give the temperatures of the globe at those particular instants of time. Taking the common logarithms of Column 2, we get Column 3; and the difference between the successive logarithms of the deflections give us Column 4. To obtain the emissivities from the numbers in Column 4, they must be multiplied by $(M \times c) / (300 \times S)$, the value of which is $3{\cdot}868 \times 10^{-3}$. This constant is smaller than that used in the 1893 paper on account of a reduction in the numerical value of c which it has been necessary to make for the diminished value of the heat capacity of the copper globes, owing to the low temperatures of the experiments.†

27. The differences of the temperatures of the globe and its surroundings are read off from the thermojunction calibration curve already mentioned, § 14, and these are placed in Column 6; and Column 7 contains the arithmetic means of the successive pairs of these numbers.

By adding the absolute temperature of the surrounding bath to the latter temperatures we get the absolute temperatures of the globe. These are placed in Column 8.

Method 2.

28. In the second method the series of observations during any one experiment, lasting perhaps $2\frac{1}{2}$ hours or more, are taken as a whole; and the emissivities at various temperatures, from the highest to the lowest, are calculated as a continuous series; the emissivity being supposed to alter continuously with the absolute temperature of the cooling body.

For purposes of numerical calculation, however, the observed deflections, and the logarithms of these numbers, are, in this method, expressed as functions of the time reckoned from the beginning of the experiment in the way which is about to be

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* In Dr. BOTTOMLEY'S 1893 experiments it was necessary to apply certain corrections to the galvanometer readings: (1) for the straight galvanometer scale used; and (2) a thermojunction correction. These were described among the details of calculation. They are not required here owing to our present improved arrangements.

† U. BEHN, 'WIEDEMANN'S Annalen,' 66, 2, pp. 237–244 (1898); 'Annalen der Physik,' 1, 2, pp. 257–269 (1900). SCHMITZ, 'Proc. Roy. Soc.,' 72, pp. 177–193 (1903).

April 2, 1906.

SILVERED Globe Cooling in Vacuum.

Vacuum = 0·06 **M.** Temperature of Enclosure = 79° abs.

	1.	2.	3.	4.	5.	6.	7.	8.
Number.	Time.	Galvano-meter deflection.	Logarithm of deflection.	Differences of logarithms.	Emissivity.	Difference of temperatures of globe and enclosure.	Mean difference of temperatures of globe and enclosure.	Absolute temperature of globe.
	A.M.					$t-t'$ ° C.	° C.	
1	10.20	867·1	2·938069			212·0		
				·003066	5·930 × 10^{-6}		210·15	289°·15
2	30	861·0	·935003			208·3		
				·002986	5·773 ,,		206·55	285·55
3	40	855·9	·932017			204·8		
				·002905	5·618 ,,		203·10	282·1
4	50	849·4	·929112			201·4		
				·003027	5·853 ,,		199·75	278·7
5	11.00	843·5	·926085			198·1		
				·002996	5·793 ,,		196·50	275·5
6	10	837·7	·923089			194·9		
				·002966	5·735 ,,		193·35	272·3
7	20	832·0	·920123			191·8		
				·002985	5·771 ,,		190·35	269·3
8	30	826·3	·917138			188·9		
				·002848	5·507 ,,		187·55	266·5
9	40	820·9	·914290			186·2		
				·002919	5·452 ,,		184·85	263·8
10	50	815·4	·911371			183·5		
				·002833	5·478 ,,		182·25	261·2
11	12.00	810·1	·908538			181·0		

12	10	804·9	·905742	·002796	5·532 „	178·7	179·85	258·8
13	20	799·7	·902927	·002815	5·431 „	176·5	177·60	256·6
14	30	794·7	·900203	·002724	5·268 „	174·4	175·45	254·4
15	40	789·8	·897517	·002686	5·194 „	172·4	173·40	252·4
16	50	785·1	·894925	·002592	5·012 „	170·5	171·45	250·4
17	1.00	780·4	·892317	·002608	5·043 „	168·8	169·65	248·6
18	10	776·1	·889862	·002455	4·748 „	167·1	167·95	246·9
19	20	771·4	·887279	·002583	4·994 „	165·5	166·30	245·3
20	30	767·0	·884795	·002484	4·803 „	163·9	163·7	243·7
21	40	762·8	·882411	·002384	4·610 „	162·5	163·2	242·2
22	50	758·6	·880013	·002398	4·637 „	161·0	161·75	240·7
23	2.00	754·6	·877717	·002296	4·440 „	159·5	160·25	239·2
24	10	750·7	·875466	·002251	4·353 „	158·2	158·85	237·8
25	20	746·9	·873263	·002203	4·260 „	156·9	157·55	236·5
26	30	743·1	·871047	·002216	4·285 „	155·7	156·30	235·3
27	40	739·5	·868938	·002109	4·079 „	154·5	155·10	234·1
28	50	735·9	·866819	·002119	4·097 „	153·4	153·95	232·9
29	3.00	732·2	·864630	·002190	4·233 „	152·3	152·85	231·8
30	10	729·0	·862728	·001902	3·680 „	151·2	151·77	230·8

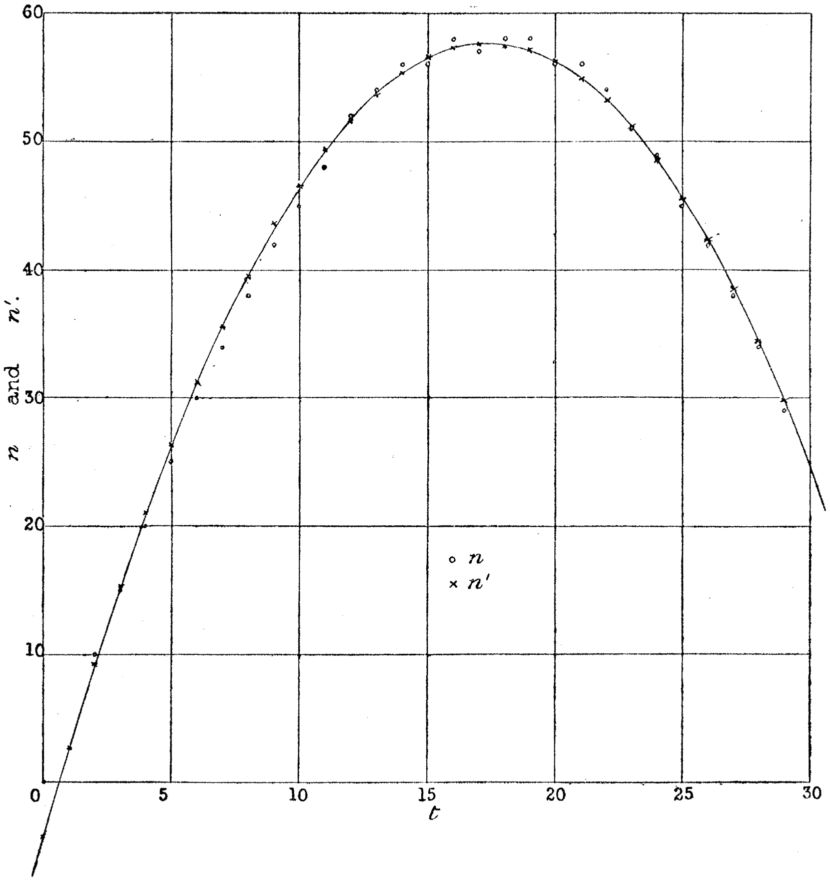
Specimen Page from Calculation Note-Book.

explained; and, the temperatures corresponding to the times being known, the emissivities can be referred to their respective temperatures.

29. Taking for an example the case of the results of April 2, 1906 (see table on p. 364, the times and the deflections (galvanometer readings) are given in Columns 1 and 3; and in Column 4 are given the logarithms of the deflections, while Column 5 contains the differences of these last, that is $\log v_0 - \log v$.* Noticing that the difference of logarithms is a number which increases as the experiment proceeds, and is a function of the time, to deal with the numbers conveniently we assume

$$\log v_0 - \log v = \mathrm{K}t + n,$$

choosing K by inspection so that Kt shall contain the greater part of the number which expresses $\log v_0 - \log v$; while n is a residue, as it were a correction on Kt, and obviously containing all the irregularities and the errors of experiment.



$n' = \alpha + \beta t + \gamma t^2$.

Silvered globe in vacuum = 0·06 M. April 2, 1906.

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* In the present investigation v_0 is the same as $(v_0 - \mathrm{V})$, and v is the same as $(v - \mathrm{V})$ of p. 358; that is, the present v_0 and v are the differences of temperatures, at times t_0 and t, between the cooling globe and the envelope.

The quantity n is now assumed to be of the parabolic form

$$n = \alpha + \beta t + \gamma t^2.$$

The justification for this assumption will be made clear presently from a comparison between observed and calculated values of n.

30. Returning, now, to the original forms, we have

$$-c\frac{dv}{dt} = e\text{S}v \quad \text{and hence} \quad e = -\frac{c}{\text{S}}\cdot\frac{1}{v}\cdot\frac{dv}{dt}.$$

But we have also

$$\log v_0 - \log v,$$

known by experiment, and equal to $\text{K}t + n$; and we have, moreover,

$$\log_\epsilon v_0 - \log_\epsilon v = \text{M}(\log v_0 - \log v),$$

where M is the modulus of the common logarithms.

Hence by differentiation, and comparison,

$$-\frac{1}{v}\frac{dv}{dt} = \frac{d}{dt}\text{M}\ (\log v_0 - \log v)$$

$$= \text{M}\frac{d}{dt}\{\text{K}t + n\} = \text{M}\ \{(\text{K}+\beta) + 2\gamma t\},$$

and, by substitution, we find

$$e = \frac{c\text{M}}{\text{S}t_1}\{(\text{K}+\beta) + 2\gamma t\};$$

t_1 (= 300) being introduced since K, α, β, γ, are in terms of 5 minutes as the unit of time.

Thus e is the emissivity when $t = 0$, plus or minus a quantity which, for want of a better name, we have been in the habit of calling "the time correction."

31. It now only remains to explain how n is dealt with, and how the values of α, β, and γ, are found. Referring again to the specimen page, on p. 364, and to Column 5, which contains the values of $\log v_0 - \log v$; the next column, Column 6, contains the values of Kt, K being chosen by inspection to be 25, so that when the values of Kt are subtracted from the corresponding numbers in the preceding Column 5, the remainders, shown in Column 7, shall be small and convenient for the process which is to follow. The numbers in Column 7 are sometimes even negative; experience directs the choosing of K. It is to be observed, also, that the factor 10^{-4} has been taken out of the numbers in Column 6. This must be kept in mind, and the factor inserted at the proper time, at the end of the calculations.

32. The next process is to form three simultaneous equations for the determination of α, β, and γ in the equation $\alpha + \beta t + \gamma t^2 = n'$. There are different ways in which this might be done, but perhaps the simplest way is to divide Column 7, containing n, into three equal parts, and construct columns containing the corresponding values of t

April 2, 1906. METHOD II.

Silvered Globe Cooling in Vacuum.

Vacuum Pressure = 0·06 **M**. Temperature of enclosure = – 194° C.

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.
t.	t^2.	Deflection.	Logarithm of deflection.	$\log v_0 - \log v$.	Kt. K = 25.	Observed. n.	Calculated. n'.	$n - n'$.	Time correction.	Emissivity.	Difference of temperatures.	Absolute temperature of globe.
											$t - t'$ °C.	°
0	0	867·1	2·9381	·0000	0	0	– 4·17	– 4·17	0	6·2103 × 10^{-6}	212·0	291·0
1	1	861·0	·9350	31	25	6	2·74	+ 3·26	·07925	6·1311 ,,	208·3	287·3
2	4	855·1	·9321	60	50	10	9·24	+ 0·76	·15850	6·0518 ,,	204·8	283·8
3	9	849·4	·9291	90	75	15	15·31	– 0·31	·23775	5·9726 ,,	201·4	280·4
4	16	843·5	·9261	120	100	20	21·0	– 1·0	·31700	5·8933 ,,	198·1	277·1
5	25	837·7	·9231	150	125	25	26·3	– 1·3	·39625	5·8141 ,,	194·9	273·9
6	36	832·0	·9201	180	150	30	31·16	– 1·16	·47550	5·7348 ,,	191·8	270·8
7	49	826·3	·9172	209	175	34	35·62	– 1·62	·55475	5·6556 ,,	188·9	267·9
8	64	820·9	·9143	238	200	38	39·66	– 1·66	·63400	5·5763 ,,	186·2	265·2
9 **45**	81 **285**	815·4	·9114	267	225	42 **220**	43·30	– 1·3	·71325	5·4971 ,,	183·5	262·5
10	100	810·1	·9086	295	250	45	46·52	– 1·52	·79250	5·4178 ,,	181·0	260·0
11	121	804·9	·9058	323	275	48	49·36	– 1·36	·87175	5·3386 ,,	178·7	257·7
12	144	799·7	·9029	352	300	52	51·74	+ 0·26	·95100	5·2593 ,,	176·5	255·5
13	169	794·7	·9002	379	325	54	53·73	+ 0·27	1·03025	5·1801 ,,	174·4	253·4
14	196	789·8	·8975	406	350	56	55·32	+ 0·68	1·10950	5·1008 ,,	172·4	251·4
15	225	785·1	·8950	431	375	56	56·49	– 0·49	1·18875	5·0216 ,,	170·5	249·5
16	256	780·4	·8923	458	400	58	57·26	+ 0·74	1·26800	4·9423 ,,	168·8	247·8
17	289	776·0	·8899	482	425	57	57·61	– 0·61	1·34725	4·8631 ,,	167·1	246·1
18	324	771·4	·8873	508	450	58	57·56	+ 0·44	1·42650	4·7838 ,,	165·5	244·5
19 **145**	361 **2185**	767·0	·8848	533	475	58 **542**	57·10	+ 0·90	1·50575	4·7046 ,,	163·9	242·9
20	400	762·8	·8825	556	500	56	56·22	– 0·22	1·58500	4·6253 ,,	162·5	241·5
21	441	758·6	·8800	581	525	56	54·94	+ 1·06	1·66425	4·5461 ,,	161·0	240·0
22	484	754·6	·8777	604	550	54	53·24	+ 0·76	1·74350	4·4668 ,,	159·5	238·5
23	529	750·7	·8755	626	575	51	51·14	– 0·14	1·82275	4·3876 ,,	158·2	237·2
24	576	746·9	·8732	649	600	49	48·62	+ 0·38	1·90200	4·3083 ,,	156·9	235·9
25	625	743·1	·8711	670	625	45	45·67	– 0·67	1·98125	4·2291 ,,	155·7	234·7
26	676	739·5	·8689	692	650	42	42·36	– 0·36	2·06050	4·1498 ,,	154·5	233·5
27	729	735·9	·8668	713	675	38	38·61	– 0·61	2·13975	4·0706 ,,	153·4	232·4
28	784	732·3	·8647	734	700	34	34·46	– 0·46	2·21900	3·9913 ,,	152·3	231·3
29 **245**	841 **6085**	729·0	·8627	754	725	29 **454**	29·90	– 0·90	2·29825	3·9121 ,,	151·3	230·3

SPECIMEN PAGE FROM CALCULATION NOTE-BOOK.

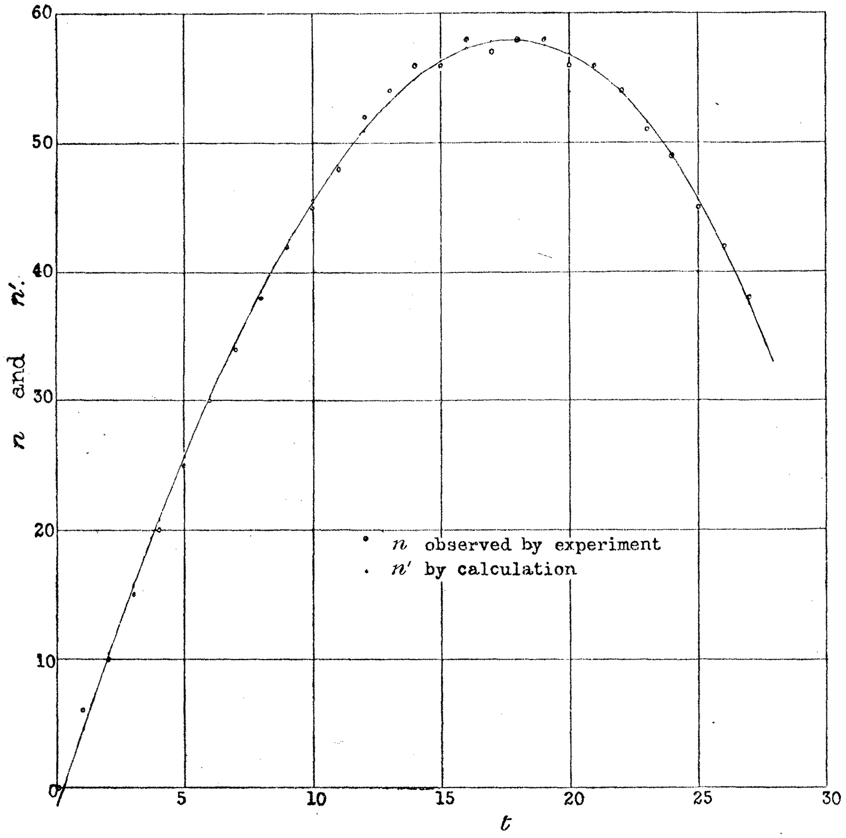
and t^2; to divide these columns similarly, then to take the sums of the numbers in each of the equal parts, and to use the respective sums as the values of n, t, and t^2, to form three equations. If this is done the equations become the following, as will be seen from the sums indicated in the specimen page, the coefficients of α being 10, as there are ten numbers in each of the three sums.

$$\begin{aligned} 10\alpha + 45\beta + 285\gamma &= 220, \\ 10\alpha + 145\beta + 2185\gamma &= 542, \\ 10\alpha + 245\beta + 6085\gamma &= 454. \end{aligned}$$

Solving these simultaneous equations, we find for the values of the three unknown quantities,

$$\alpha = -4{\cdot}175\,;\quad \beta = 7{\cdot}115\,;\quad \gamma = -0{\cdot}205.$$

33. Having determined the values of α, β, and γ, the next process is to find the series of values of n', which is done by putting the numerical values of t into the equation $n' = \alpha + \beta t + \gamma t^2$. The number n' is not required for the calculation of e, but is needed for comparison with n, and for verification of the exactness of the process. The values of βt and γt^2 are most easily obtained by means of CRELLE's Multiplication



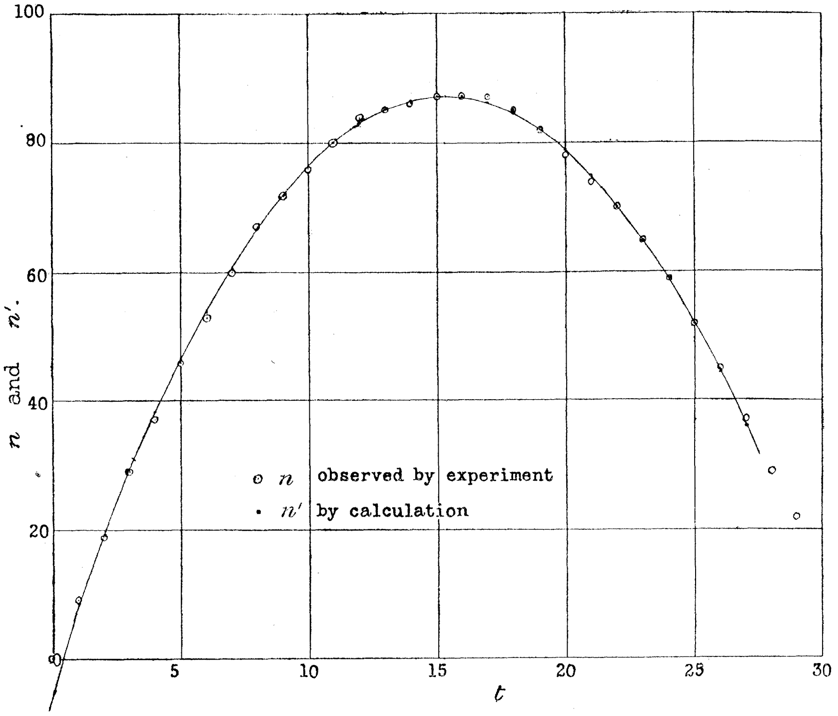
$n' = \alpha + \beta t + \gamma t^2 + \delta t^3.$

Silvered globe in vacuum = 0·06 M. April 2, 1906.

[In this diagram and the next one the small dots representing n' have not come out, as they lie on the curve.]

Tables. When calculated they are placed in suitable columns not shown in the specimen table, and from these columns the successive values of n' are obtained by addition, and they are placed in Column 8; while Column 9 gives the comparison between n and n'. Remembering that $n-n'$ has to be multiplied by 10^{-4}, it will be seen that the process of averaging leaves nothing to be desired.

34. An interesting experimental way of looking at this last question is to plot the values of n and n' on squared paper and to compare the curves. The results are shown in the accompanying diagrams, pp. 362, 365, 366; and the closeness of the agreement between the smooth and the unsmoothed curves is shown to the eye to be quite satisfactory. If it were not, the necessity for revision of the work would at once be revealed. In every case the plotted form of n and n' has been found to be obviously parabolic.



$n' = \alpha+\beta t+\gamma t^2+\delta t^3.$

Sooted globe cooling in vacuum. September 18, 1906.

35. Column 10 contains the "time correction," as defined above. The numbers in this column are added to the emissivity at $t = 0$, which is the number that heads Column 11, and the sums thus obtained give the emissivities at times $t = 1$, $t = 2$, &c., &c. These emissivities are placed in Column 11; and opposite to them, in Column 12, the differences of temperatures between cooling body and surroundings at those times are shown; while, in Column 13, the corresponding absolute temperatures of the cooling globe are given.

Method 2*a*.

36. In some cases it was found desirable to use a fourth term, containing t^3, in the expression for n', when the formula stopping with t^2 did not give sufficient accuracy. This involved the finding of a fourth constant, δ. In this case $n' = \alpha + \beta t + \gamma t^2 + \delta t^3$.

To calculate the emissivities the procedure is similar to that just described, except that four equations are formed for the determination of α, β, γ, δ; and by suitable modifications in the equations, p. 363, we find

$$e = \frac{\mathrm{M}c}{\mathrm{S}t_1}\{\mathrm{K} + \beta + (2\gamma t + 3\delta t^2)\};$$

the time correction, instead of being a linear quantity, containing a squared term. In most cases δ was negligible, and the first form of n' was found to be quite sufficient for our purpose.

Discussion of Results.

37. The following tables contain the results of our experiments on radiation of heat at low temperatures. The tables are divided into three groups. The first group, Tables I. to VI. and XI., gives the loss of heat from a copper globe covered with a very fine coating of soot, hung in a spherical copper shell, also sooted, and at the highest vacuum. In this group, therefore, we have probably the nearest practicable approach to the case of a perfectly black body cooling in vacuum at very low temperatures.

38. In the second group, Tables VII. to X. and XII., all the circumstances are practically the same as in the first, except that the surface of the copper globe was covered with a coating of silver, polished to the highest degree to which we could attain.

39. The third group, Tables XIII. to XV., gives the cooling of a sooted globe, and of a highly polished silvered globe, in the same enclosure as is specified above, but in air at about standard pressure, and all at very low temperatures.

40. Tables I. and II. show the radiation from the sooted globe, at the highest vacuums we could reach, the temperature of the cooling globe being at the commencement about 10° C., and falling during the experiment, which lasted $2\frac{1}{2}$ hours, to $-59°$ C., the envelope being maintained at the temperature of liquid air during the whole time. These two tables are placed side by side in order that it may be seen to what extent agreement was found between experiments made at different dates, the circumstances being, as far as possible, repeated. The comparison shows that the exactness of agreement leaves nothing to be desired. Whatever there is of difference between the numbers obtained for the emissivity is certainly to be attributed to difference in the condition as to vacuum. This will be seen from comparisons among themselves of experiments at different vacuums.

41. It should be noted here that the vacuums attained in these experiments are certainly of a very much higher order than those obtained in Dr. Bottomley's older

experiments on this subject. The presence of the liquid air surrounding the enclosure causes condensation of any kind of collapsible vapour which might exist in the so-called vacuous space, *e.g.*, vapour of mercury, or any trace of vapour of water, &c., the existence of which would not be indicated by the M'Leod gauge, but which, should it exist, would certainly play its part in causing heat transference by convection.

42. Tables III., IV., V. give the emissivity from a sooted globe at a good vacuum, but less perfect than in the cases of Tables I. and II., and at gradually increasing pressure up to 60 **M.**, or about half-a-tenth of a millimetre.

43. The construction of the tables is clearly indicated by the headings. For each experiment there is given the condition of the surface, the vacuum pressure, and the temperature of the enclosure. In the second column is given the difference of temperatures between the globe and its surroundings at the time of the galvanometer reading. This difference, as has been stated, is obtained from the calibration curves of the thermojunctions. Column 3 gives the emissivity, calculated as has been explained and exemplified in Section 5 of the paper. Column 4 gives the absolute temperature of the globe at the moment of obtaining the galvanometer reading. Column 5 will be explained below. It contains what has been called "the radiation constant," calculated in accordance with STEFAN'S law.

44. Coming now to the tables of the second group, these give the emissivity for the highly polished silvered globe. Tables VII. and VIII. show the loss of heat at the highest vacuum we could obtain, with Sprengel pumps and cooled charcoal; and it will be seen that the loss from the sooted surface under these circumstances is four times as much as from the highly polished silvered surface at the same absolute temperatures of the cooling body and surroundings. This result quite agrees with those obtained in the earlier experiments of Dr. J. T. BOTTOMLEY.

Tables IX. and X. give the loss from a silvered surface in a less perfect vacuum. These, however, are of comparatively small interest.

45. Tables VI., XI., and XII., give the case of the silvered globe and of the sooted globe cooled to a temperature below that of the enclosure, and receiving heat by radiation from warmer surroundings.

Not many experiments of this kind have been made, so far as we are aware; but the results are interesting when considered in connection with the principle of "Heat Exchanges."

It may be noticed that in every case, both sooted and silvered, where the globe has been cooled below the temperature of its surroundings, and is allowed to rise in temperature by receiving heat from the walls of the enclosure, the calculated emissivity has turned out to be higher than in the reverse, and more ordinary, experiment. It is probable that this is due to deterioration of the vacuum, on account of the escape of gases or vapours from the surrounding walls on the removal of the liquid air. Such deterioration of the vacuum would not be indicated by the M'Leod gauge, as there is always a very considerable time lag in the readings of the gauge. Vacuum is very slowly transmitted from point to point along the connecting tubes which must intervene between the gauge and the space whose condition, as to vacuum, the gauge is to measure. In a foot-note, p. 352, this is referred to. The connecting tubes have been made as short as possible, but in spite of every care in this direction this inherent trouble has not been overcome.

46. The third group contains three tables which show the cooling of the copper globe in air at ordinary atmospheric pressure. In Table XIII. the copper globe is finely sooted; in Table XIV. the surface of the globe is silvered and brilliantly polished; while Table XV. gives the cooling of the globe with a silvered, but dull white surface.*

A comparison of these tables with the corresponding tables for the globes cooling in vacuum shows the part which is played by convection in the ordinary case of bodies cooling in air.

47. In the following short tables, the results of such a comparison for the cases of the sooted globe and the highly polished silvered globe are given. The emissivities† have been taken at corresponding temperatures for the globe cooling in vacuum, and cooling in air, for the two cases mentioned. These are placed opposite the temperatures to which they belong, and Column 1 of each table gives the absolute temperatures of the cooling globe. Column 2 gives the emissivity in vacuum Column 3 gives the number which corresponds to the emissivity in air; while Column 4 gives the ratios between the numbers in Column 3 and those in Column 2. The following points may be noticed with regard to the numbers in these tables. In the first place the relatively high numbers for the emissivity in air are very remarkable, and it is to be noticed that the numbers are very nearly the same for the sooted globe and the silvered globe. The true radiation goes almost for nothing in comparison with the loss due to radiation and convection combined.

1.	2.	3.	4.	1.	2.	3.	4.
Sooted globe.				Silvered globe.			
Absolute temperature.	e_1 in vacuum.	e_2 in air.	Ratio $\frac{e_2}{e_1}$.	Absolute temperature.	e_1 in vacuum.	e_2 in air.	Ratio $\frac{e_2}{e_1}$.
° C.				° C.			
284	$2 \cdot 152 \times 10^{-5}$	$3 \cdot 842 \times 10^{-4}$	17·85	270	$5 \cdot 368 \times 10^{-6}$	$3 \cdot 659 \times 10^{-4}$	68·2
260	$1 \cdot 948 \times 10^{-5}$	$3 \cdot 829 \times 10^{-4}$	19·6	260	$5 \cdot 045 \times 10^{-6}$	$3 \cdot 651 \times 10^{-4}$	72·4
240	$1 \cdot 714 \times 10^{-5}$	$3 \cdot 783 \times 10^{-4}$	22·1	240	$4 \cdot 115 \times 10^{-6}$	$3 \cdot 637 \times 10^{-4}$	88·4
220	$1 \cdot 393 \times 10^{-5}$	$3 \cdot 736 \times 10^{-4}$	26·8	230	$3 \cdot 501 \times 10^{-6}$	$3 \cdot 631 \times 10^{-4}$	103·4

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* This surface was caused by a fine deposit of mercury on the highly polished silvered surface.

† It is not in accordance with strict language to use the word "emissivity" in the case where air is present; but the lapse from strictness may be pardoned here for the sake of brevity.

48. The results of experiments made at the beginning of last century by Sir JOHN LESLIE and others seemed to show that the total loss of heat from a body cooling in dry air was made up of radiation and convection in about equal proportions. Such a statement as this is, however, very far from representing the state of the case with regard to loss of heat with full air pressure at very low temperatures. The loss by convection is enormously increased when the air is reduced in temperature nearly to liquefying point. On the other hand, the loss by pure radiation from the sooted globe at very low temperatures is extremely small; while the loss from the highly polished silvered globe is, so to speak, minute. Thus convection may be from 18 to 25 times the pure radiation from the sooted surface, instead of being about equal to it; while the convection is from 60 to 100 times the pure radiation from polished silver. We propose shortly to make some special experiments on this subject.

49. A question of great interest is the comparison of the results we have obtained, including Dr. BOTTOMLEY'S old results, with the 4th power law, or formula of STEFAN. According to STEFAN'S law, the emission, S, of heat from a "black" body is proportional to the 4th power of the absolute temperature of the cooling surface. Taking this law in conjunction with the law of "heat exchanges," the cooling of a "black" body at temperature θ in a "black" enclosure at temperature θ_0 ought to be proportional to the product of the "emission" and $(\theta^4-\theta_0^4)$; and may be represented by $\sigma(\theta^4-\theta_0^4)$, where σ is a constant, sometimes called "the radiation constant." In our tables it is not the "emission" of heat per square centimetre which is given, but the "emissivity," or the emission divided by the difference of temperatures between the cooling body and the surrounding envelope. If, then, e be the emissivity between θ and θ_0, we have $e = \mathrm{S}/(\theta-\theta_0)$; and, correspondingly, we shall have

$$\sigma = \frac{\mathrm{S}}{(\theta^4-\theta_0^4)} = \frac{e}{(\theta+\theta_0)(\theta^2+\theta_0^2)}.$$

We have applied this formula to all the tables where the cooling of the sooted globe is given, and where the vacuum is sufficiently good to make the experiment suitable for the purpose in hand. These numbers are given in Column 5 of the respective tables.

TABLES taken from 1893 Paper of J. T. BOTTOMLEY, for Comparison of Emissivities, and of STEFAN'S 4th Power Law.

	Oct. 22, 1889. Vacuum = 0·77 **M.** θ_0 Temp. of enclosure = 290°·5 abs.			Oct. 29, 1889. Vacuum = 0·45 **M.** θ_0 Temp. of enclosure = 288°·3 abs.			April 2, 1890. Vacuum = 0·8 **M.** θ_0 Temp. of enclosure = 289° abs.			April 8, 1890. Vacuum = 0·2 **M.** θ_0 Temp. of enclosure = 288° abs.		
	Emissivity.	Absolute temp. of globe.	Emission / $\theta^4 - \theta_0^4$.	Emissivity.	Absolute temp. of globe.	Emission / $\theta^4 - \theta_0^4$.	Emissivity.	Absolute temp. of globe.	Emission / $\theta^4 - \theta_0^4$.	Emissivity.	Absolute temp. of globe.	Emission / $\theta^4 - \theta_0^4$.
		θ			θ			θ			θ	
1	1·37 × 10^{-4}	370°·1	9·373 × 10^{-13}	1·25 × 10^{-4}	371°·9	8·551 × 10^{-13}	2·78 × 10^{-4}	512°·0	10·04 × 10^{-13}	2·23 × 10^{-4}	505°·5	8·30 × 10^{-13}
2	1·36 ,,	364·5		1·23 ,,	366·5		2·02 ,,	484·9		2·00 ,,	482·3	8·23 ,,
3	1·34 ,,	359·4	9·655 ,,	1·21 ,,	361·6	8·705 ,,	1·96 ,,	465·2	8·66 ,,	1·78 ,,	456·1	8·22 ,,
4	1·32 ,,	354·7		1·19 ,,	356·0		1·89 ,,	448·0		1·63 ,,	433·8	8·33 ,,
5	1·30 ,,	350·4	9·791 ,,	1·17 ,,	352·9	8·787 ,,	1·82 ,,	433·0	9·32 ,,	1·52 ,,	422·1	8·20 ,,
6	1·29 ,,	346·4		1·15 ,,	349·0		1·76 ,,	419·8		1·44 ,,	411·9	8·15 ,,
7	1·27 ,,	342·7	9·937 ,,	1·14 ,,	345·4	8·887 ,,	1·72 ,,	408·3	9·83 ,,	1·42 ,,	402·8	8·38 ,,
8	1·25 ,,	339·3		1·12 ,,	341·2		1·61 ,,	403·2		1·36 ,,	394·6	8·35 ,,
9	1·23 ,,	336·2	9·942 ,,	1·10 ,,	339·0	8·855 ,,	1·57 ,,	389·3	9·82 ,,	1·27 ,,	387·4	8·07 ,,
10	1·22 ,,	333·3		1·08 ,,	336·1		1·50 ,,	381·5		1·29 ,,	380·9	8·46 ,,
11	1·20 ,,	330·6	9·975 ,,	1·06 ,,	333·4	8·776 ,,	1·43 ,,	374·5	9·64 ,,	1·20 ,,	374·9	8·10 ,,
12	1·18 ,,	328·2		1·05 ,,	331·0		1·37 ,,	368·4		1·22 ,,	369·4	8·58 ,,
13	1·16 ,,	325·9	9·874 ,,	1·03 ,,	328·7	8·733 ,,	1·30 ,,	362·9	9·27 ,,	1·20 ,,	364·4	8·53 ,,
14	1·13 ,,	321·8		0·99 ,,	324·6		1·24 ,,	358·0	9·03 ,,	1·26 ,,	359·6	9·17 ,,
15	1·09 ,,	318·3	9·641 ,,	0·95 ,,	321·0	8·376 ,,				1·15 ,,	355·1	8·56 ,,
16	1·06 ,,	315·3								1·07 ,,	351·2	8·12 ,,
17	1·02 ,,	312·7	9·282 ,,									
18	0·989 ,,	310·5										
19	0·954 ,,	308·5	8·870 ,,									

NOTE.—*By Dr.* J. T. BOTTOMLEY.

Added July 11, 1907.—The foregoing paper contains an account of experiments on radiation of heat at very low temperatures, carried out by Mr. F. A. KING and the writer of the present note. From the results of these experiments we have calculated the thermal emissivity, from a carefully sooted surface, at the temperature stated. An attempt is made at the end of the paper to put these results into relation with the results of older experiments which I carried out fourteen years ago, with the same apparatus, and in precisely the same way, but at considerably higher temperatures. No comparison was, however, made in my former papers on this subject between the results of these experiments and what I may call the theoretical laws of thermal radiation, nor with the law of STEFAN deduced from the old experiments of DULONG and PETIT. The matter was entirely dealt with from an experimental point of view.

It seems desirable, now, that I should make an endeavour to compare our experimental results, as far as possible, both with STEFAN'S law and with results more recently obtained by other experimenters.

Careful experiments on thermal radiation have been made by WIEN and LUMMER, E. ST. JOHN, C. CHRISTIANSEN, LUMMER and PRINGSHEIM, and by F. KURLBAUM; but I find it very difficult to compare their results with my own. All my experiments have been made by the method of cooling, which gives directly, in absolute measure, the emissivity of the surface of the heated body. Almost all the other experimenters have used an indirect method, and have inferred the radiating power of the heated body from the indications of a bolometer or special form of thermojunction, determining the temperature to which this receptor is raised by the presence of the radiating body. In a paper by KURLBAUM[*] the experiments were carried on by means of the bolometer. The radiation was from the theoretical "black body" used by WIEN and LUMMER[†], *i.e.*, from the blackened interior of a hollow vessel, heated externally with steam, and having an opening in front through which the radiation passes; and the bolometer, placed opposite to the opening, receives the radiation. The transition from arbitrary to absolute units is difficult. The method is to find the temperature to which the bolometer rises, and at which it permanently stands, when at a given distance from the heated radiating body, and then to ascertain the amount of electric current which will keep the bolometer at this temperature. It seems difficult to put these two observations into sure relationship with each other, and to obtain the radiating power of the black body in absolute measure from experiments on the heat received by the cooler of the two bodies; but it is very interesting to find a close agreement between the results obtained by this indirect method and those obtained simply by directly measuring the

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[*] 'WIEDEMANN'S Annalen,' No. 65, 1898.

[†] 'WIEDEMANN'S Annalen,' No. 56, 1895, p. 451.

loss of heat from the warm radiator, particularly when it is considered how minute a proportion of the heat radiated in all directions falls upon the bolometer and is utilised for the measurement.

KURLBAUM also quotes in his paper the results of earlier investigators, but these experiments also are mostly indirect. Thus the results of LEHNEBACH are deduced with the help of a comparison between the radiating power of glass and that of the theoretical black body; and the same applies to those of GRAETZ. A result deduced from the work of KUNDT and WARBURG depends on the conducting power of gases for heat, while numbers quoted from CHRISTIANSEN are founded on considerations still more complicated.

In each case KURLBAUM deduces the emission (not emissivity) of heat by a body at 100° C. to a body at 0° C., or rather the heat received by a body at 0° C. from a "black" surface at 100° C. placed opposite to it. This is taken as the "emission" of heat between the absolute temperatures 273° C. and 373° C., and STEFAN'S coefficient is deduced from these numbers.

The following table is taken from KURLBAUM'S paper, except that I have interpolated Column 3. The results are in Gm. Cal./cm^2 × sec. :—

Observer.	$h_{100}-h_0$.	Average emissivity between 0° C. and 100° C.	STEFAN'S coefficient. σ.
LEHNEBACH, 1874	0·0152	$1\cdot52\times10^{-4}$	$11\cdot0\times10^{-13}$
KUNDT and WARBURG, 1875	0·014	$1\cdot4\ \times10^{-4}$	$10\cdot1\times10^{-13}$[*] $11\cdot1\times10^{-13}$
GRAETZ, 1880	0·0150	$1\cdot5\ \times10^{-4}$	$10\cdot8\times10^{-13}$
CHRISTIANSEN, 1883	0·0167	$1\cdot67\times10^{-4}$	$12\cdot1\times10^{-13}$
KURLBAUM, 1898	0·0176	$1\cdot76\times10^{-4}$	$12\cdot8\times10^{-13}$[†]

In calculating the results of my experiments, following my original plan, I am not led to tabulate "emissions" but "emissivities." Hence my figures do not compare directly with those placed in the column of this table under the heading $h_{100}-h_0$. For this reason I have interpolated Column 3, which shows what may be taken to be the average *emissivity* between the temperatures 100° C. and 0° C. For comparison with these numbers I may quote the following table, compiled from my paper in the 'Phil. Trans.' for 1893. The numbers are taken out of tables giving emissivities at different, but extremely low, pressures.

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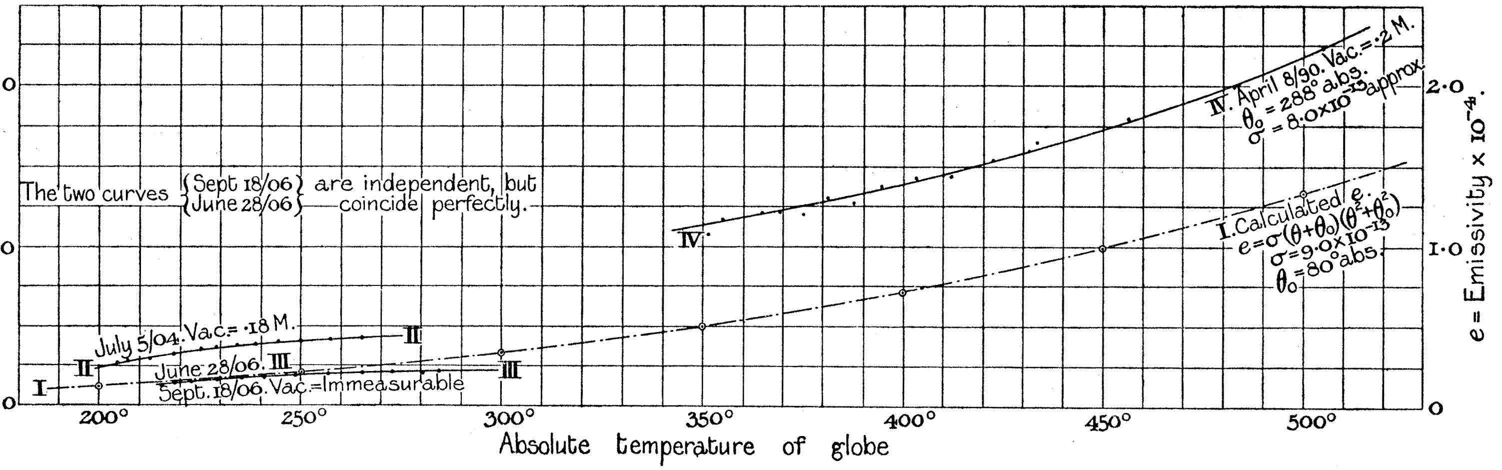
[*] The number 10·1 is reckoned by using the value 0·014 as the emission between $h_{100}-h_0$, as given above; 11·1 is a number obtained by GRAETZ from the data given by KUNDT and WARBURG.

[†] This number seems to be considered as final by KURLBAUM. I find that he makes use of it in subsequent papers as an accepted figure, *e.g.*, 'WIED. Ann.', Band 2, 1900, p. 550; and others quote it as authoritative. See, *e.g.*, POYNTING, 'Phil. Trans.', 1904, p. 526, and POYNTING and THOMSON, 'Heat.'

Date.	Vacuum.	Temperature of globe.	Temperature of enclosure.	Emissivity (defined).	Emission divided by $(\theta^4 - \theta_0^4)$.
	M.	° C.	° C.		
October 22, 1889 . .	0·77	97·0	17·5	$1 \cdot 37 \times 10^{-4}$	$9 \cdot 373 \times 10^{-13}$
		63·0		$1 \cdot 23 \times 10^{-4}$	$9 \cdot 94 \times 10^{-13}$
		45·0		$1 \cdot 09 \times 10^{-4}$	$9 \cdot 64 \times 10^{-13}$
October 29, 1889 . .	0·45	99·0	15·3	$1 \cdot 25 \times 10^{-4}$	$8 \cdot 55 \times 10^{-13}$
		72·0		$1 \cdot 14 \times 10^{-4}$	$8 \cdot 89 \times 10^{-13}$
		56·0		$1 \cdot 03 \times 10^{-4}$	$8 \cdot 73 \times 10^{-13}$
April 2, 1890 . . .	0·8	116·0	16·0	$1 \cdot 57 \times 10^{-4}$	$9 \cdot 82 \times 10^{-13}$
		101·5		$1 \cdot 43 \times 10^{-4}$	$9 \cdot 64 \times 10^{-13}$
		85·0		$1 \cdot 24 \times 10^{-4}$	$9 \cdot 03 \times 10^{-13}$
April 8, 1890 . . .	0·2	96·4	15·0	$1 \cdot 22 \times 10^{-4}$	$8 \cdot 58 \times 10^{-13}$
		86·6		$1 \cdot 26 \times 10^{-4}$	$9 \cdot 17 \times 10^{-13}$
		78·2		$1 \cdot 07 \times 10^{-4}$	$8 \cdot 12 \times 10^{-13}$

From these tables, given complete on p. 371, I have extracted numbers corresponding to temperatures between the limits 0° C. and 100° C. It will be seen that the values I have obtained for STEFAN'S coefficient are also lower than those which they have obtained at similar temperatures; but my numbers do not differ so much from the results of GRAETZ and KUNDT and WARBURG as do those of CHRISTIANSEN and KURLBAUM from the results of the last-named observers.

The accompanying sheet of curves brings out some of the relations of the numbers under consideration in a somewhat interesting way. Taking the Stefan coefficient to be 9×10^{-13}, and taking the temperature of a sooted enclosure such as I used in my experiments to be 80° absolute, I have calculated corresponding emissivities; and on plotting these they form curve I. on the sheet. To compare these with emissivities obtained by experiment, I have laid down the results of the experiments of Mr. KING and myself of dates July 5, 1904, and June 28, September 18, 1906. In each case the surrounding envelope was at the temperature of liquid air. In one case the vacuum was 0·18 M., and in the other it was absolutely unmeasurable, and as nearly perfect as can be obtained by the use of charcoal and liquid air. I have also traced the curve for my old results of April 8, 1890, where the temperature of the enclosure was 15° C., or 288° absolute, and the vacuum was 0·2 M. Now comparing the curves for July 5 and September 18, it will be seen that the emissivity in the case of the former at 0·18 M. is rather more than double that at the nearly perfect vacuum; and it seems not unnatural to suppose that, if I had been able to improve correspondingly the vacuum of April 8, the emissivity would have been reduced to less than half of that which I found. I have other evidence that this supposition is very close to the truth. If the circumstances under which this 1890 experiment was carried out had been precisely comparable with those represented by curves II. and III., it seems almost sure that the emissivity found would have been very close to that shown on curve I.



At the present time, Mr. KING and I are endeavouring to make arrangements for carrying out experiments on the cooling of a sooted body, very much on the same lines as those on which we have been working, but with the cooling globe heated at the commencement to a temperature considerably above that of boiling water, and with the enclosure kept all the time at the temperature of liquid air. In our experiments up till now we have only been able to work, as it were, piecemeal and in sections; first, from the high temperature down to say 15° C., and then from 15° on to the lower temperature of liquid air. An experiment through the long range, and with the enclosure kept at the temperature of liquid air, would give data suitable for direct comparison with curve III.

It may be, however, that after all a copper globe sooted is not a true "black" body, no matter how carefully prepared. One point, among others, on which there seems great uncertainty in this connection, is the question of conduction of heat from the surface of the copper to the sooty covering. This was a difficulty with the late Sir GEORGE STOKES, who discussed it with me on more than one occasion.

I am preparing comparative experiments to test my sooted globe against the artificial black body.

DETAILS OF SEPARATE EXPERIMENTS.

GROUP I.—Sooted Globe Cooling in Vacuum.

I.
September 18, 1906.
Copper globe finely sooted.*
Vacuum pressure immeasurable on M'Leod gauge.
Temperature of enclosure = 79°·5 absolute.

No.	Difference of temperatures. $(t-t')$° C.	Emissivity. e.	Absolute temperature of globe. θ.	Emission $\frac{}{\theta^4-\theta_0^4}$.
	° C.		°	
1	193·1	2·064 × 10^{-5}	272·6	7·270 × 10^{-13}
2	189·6	2·035 ,,	269·1	7·414 ,,
3	183·1	1·976 ,,	262·6	7·673 ,,
4	174·3	1·889 ,,	253·8	8·012 ,,
5	166·6	1·802 ,,	246·1	8·274 ,,
6	159·7	1·714 ,,	239·2	8·484 ,,
7	153·6	1·626 ,,	233·1	8·575 ,,
8	148·2	1·539 ,,	227·7	8·613 ,,
9	143·1	1·452 ,,	222·6	8·603 ,,
10	138·7	1·363 ,,	218·2	8·489 ,,

II.
June 28, 1906.
Copper globe finely sooted.
Vacuum pressure = 0·03 **M.**
Temperature of enclosure = 79° absolute.

No.	Difference of temperatures. $(t-t')$° C.	Emissivity. e.	Absolute temperature of globe. θ.	Emission $\frac{}{\theta^4-\theta_0^4}$.
	° C.		°	
1	194·9	2·021 × 10^{-5}	273·9	7·047 × 10^{-13}
2	181·5	1·922 ,,	260·5	7·640 ,,
3	170·6	1·824 ,,	249·6	8·098 ,,
4	165·8	1·775 ,,	244·8	8·284 ,,
5	161·2	1·726 ,,	240·2	8·456 ,,
6	153·0	1·627 ,,	232·0	8·710 ,,
7	149·3	1·578 ,,	228·3	8·799 ,,
8	142·4	1·480 ,,	221·4	8·918 ,,
9	139·4	1·430 ,,	218·4	8·914 ,,
10	136·5	1·381 ,,	215·5	8·901 ,,

III.
July 5, 1904.
Copper globe sooted.
Vacuum pressure = 0·18 **M.**
Temperature of enclosure = 80° absolute.

No.	Difference of temperatures. $(t-t')$° C.	Emissivity. e.	Absolute temperature of globe. θ.	Emission $\frac{}{\theta^4-\theta_0^4}$.
	° C.		°	
1	185·2	4·217 × 10^{-5}	265·2	15·92 × 10^{-13}
2	170·0	3·985 ,,	250·0	17·53 ,,
3	164·2	3·870 ,,	244·2	18·08 ,,
4	158·7	3·755 ,,	238·7	18·68 ,,
5	154·0	3·639 ,,	234·0	18·95 ,,
6	145·4	3·407 ,,	225·4	19·50 ,,
7	138·2	3·176 ,,	218·2	19·72 ,,
8	132·3	2·945 ,,	212·3	19·57 ,,
9	127·3	2·713 ,,	207·3	19·13 ,,
10	124·7	2·598 ,,	204·7	18·89 ,,

IV.
June 21, 1906.
Copper globe finely sooted.
Vacuum pressure = 8·0 **M.**†
Temperature of enclosure = 80° absolute.

No.	Difference of temperatures. $(t-t')$° C.	Emissivity. e.	Absolute temperature of globe. θ.	Emission $\frac{}{\theta^4-\theta_0^4}$.
	° C.		°	
1	201·1	4·827 × 10^{-5}	281·1	
2	192·5	4·767 ,,	272·5	
3	184·8	4·707 ,,	264·8	
4	171·0	4·588 ,,	251·0	
5	158·9	4·469 ,,	238·9	
6	153·3	4·409 ,,	233·3	
7	143·4	4·290 ,,	223·4	
8	134·7	4·170 ,,	214·7	
9	127·2	4·052 ,,	207·2	
10	123·7	3·992 ,,	203·7	

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* Copper globe with same coating of soot as in experiment of June 28, 1906.

† Copper enclosure leaking slightly.

Group I.—Sooted and Silvered Globes Cooling in Vacuum (continued).

V.
May 31, 1905.
Copper globe sooted.
Vacuum pressure = 60 **M**.*
Temperature of enclosure = 81° absolute.

No.	Difference of temperatures. $(t-t')$° C.	Emissivity. *e*.	Absolute temperature of globe. θ.
	° C.		
1	209·6	$8\cdot914 \times 10^{-5}$	290°·6
2	165·4	8·559 ,,	246·4
3	133·0	8·029 ,,	214·0
4	118·6	7·675 ,,	199·6
4	107·0	7·322 ,,	188·0
6	97·8	6·968 ,,	178·8
7	86·3	6·438 ,,	167·3
8	76·5	5·908 ,,	157·5
9	68·1	5·378 ,,	149·1
10	63·3	5·023 ,,	144·3

VI.
September 18, 1906.
Copper globe finely sooted.
Increasing in temperature.
Vacuum pressure immeasurable on M'Leod gauge.
Temperature of enclosure = 291°·8 absolute.

No.	Difference of temperatures. $(t-t')$° C.	Emissivity. *e*.	Absolute temperature of globe. θ.
	° C.		
1	60·6	$3\cdot742 \times 10^{-5}$	231°·2
2	56·2	3·947 ,,	235·6
3	52·1	4·153 ,,	239·7
4	47·9	4·358 ,,	243·9
5	44·1	4·564 ,,	247·7
6	40·4	4·769 ,,	251·4
7	36·8	4·975 ,,	255·0
8	33·4	5·181 ,,	258·4
9	30·3	5·386 ,,	261·7
10	25·7	5·728 ,,	266·1

XI.
July 5, 1904.
Copper globe sooted.
Increasing in temperature.
Vacuum pressure = 1·8 **M**.
Temperature of enclosure = 287° absolute.

No.	Difference of temperatures. $(t-t')$° C.	Emissivity. *e*.	Absolute temperature of globe. θ.
	° C.		
1	75·8	$6\cdot092 \times 10^{-5}$	211°·2
2	70·2	6·261 ,,	216·8
3	64·7	6·430 ,,	222·3
4	59·6	6·597 ,,	227·4
5	54·8	6·766 ,,	232·2
6	50·0	6·934 ,,	237·0
7	45·7	7·103 ,,	241·3
8	41·7	7·271 ,,	245·3
9	37·7	7·440 ,,	249·3
10	31·0	7·778 ,,	256·0

XII.
July 21, 1905.
Silvered globe highly polished.
Increasing in temperature.
Vacuum pressure = 0·1 **M**.
Temperature of enclosure = 294° absolute.

No.	Difference of temperatures. $(t-t')$° C.	Emissivity. *e*.	Absolute temperature of globe. θ.
	° C.		
1	16·9	$9\cdot206 \times 10^{-6}$	277°·1
2	16·7	9·296 ,,	277·3
3	16·5	9·382 ,,	277·5
4	16·3	9·470 ,,	277·7
5	16·1	9·556 ,,	277·9
6	15·95	9·645 ,,	278·05
7	15·75	9·734 ,,	278·25
8	15·38	9·908 ,,	278·62
9	15·0	10·083 ,,	279·0
10	14·61	10·258 ,,	279·39

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* The copper enclosure was leaking slightly and consequently the vacuum pressure was variable. The average pressure was 60 **M**.

GROUP II.—Silvered Globe Cooling in Vacuum.

VII.
March 23, 1906.
Silvered globe highly polished.
Vacuum pressure = 0·05 **M.**
Temperature of enclosure = 81° absolute.

No.	Difference of temperatures. $(t - t')°$ C.	Emissivity. *e*.	Absolute temperature of globe. *θ*.
	° C.		
1	207·3	$5{\cdot}812 \times 10^{-6}$	288°·3
2	197·3	5·581 ,,	278·3
3	188·4	5·349 ,,	269·4
4	181·0	5·118 ,,	262·0
5	174·7	4·887 ,,	255·7
6	169·1	4·655 ,,	250·1
7	164·4	4·424 ,,	245·4
8	160·1	4·191 ,,	241·1
9	156·2	3·960 ,,	237·2
10	150·3	3·565 ,,	231·3

Globe has not been polished since July 20, 1905. In the meantime it has been hanging within the enclosure, which was exhausted.

VIII.
April 2, 1906.
Silvered globe highly polished.
Vacuum pressure = 0·06 **M.**
Temperature of enclosure = 79° absolute.

No.	Difference of temperatures. $(t - t')°$ C.	Emissivity. *e*.	Absolute temperature of globe. *θ*.
	° C.		
1	212·0	$6{\cdot}210 \times 10^{-6}$	291°·0
2	201·4	5·972 ,,	280·4
3	191·8	5·734 ,,	270·8
4	183·5	5·497 ,,	262·5
5	176·5	5·259 ,,	255·5
6	170·5	5·021 ,,	249·5
7	165·5	4·783 ,,	244·5
8	161·0	4·545 ,,	240·0
9	156·9	4·308 ,,	235·9
10	152·3	3·991 ,,	231·3

IX.
July 20, 1905.
Silvered globe very highly polished.
Vacuum pressure = 0·1 **M.**
Temperature of enclosure = 79° absolute.

No.	Difference of temperatures. $(t - t')°$ C.	Emissivity. *e*.	Absolute temperature of globe. *θ*.
	° C.		
1	211·0	$4{\cdot}027 \times 10^{-6}$	290°·0
2	207·5	3·990 ,,	286·3
3	203·7	3·953 ,,	282·7
4	200·4	3·917 ,,	279·4
5	197·0	3·879 ,,	276·0
6	193·8	3·842 ,,	272·8
7	190·7	3·805 ,,	269·7
8	187·8	3·768 ,,	266·8
9	184·2	3·719 ,,	263·2
10	181·0	3·670 ,,	260·0

Globe repolished before this experiment.

X.
July 11, 1905.
Silvered globe tarnished surface.
Vacuum pressure = 20 **M.**
Temperature of enclosure = 79° absolute.

No.	Difference of temperatures. $(t - t')°$ C.	Emissivity. *e*.	Absolute temperature of globe. *θ*.
	° C.		
1	215·0	$5{\cdot}178 \times 10^{-5}$	294°·0
2	175·3	5·022 ,,	254·3
3	151·3	4·867 ,,	230·3
4	139·6	4·764 ,,	218·6
5	134·6	4·713 ,,	213·6
6	121·3	4·557 ,,	200·3
7	110·5	4·402 ,,	189·5
8	101·4	4·247 ,,	180·4
9	93·5	4·092 ,,	172·5
10	76·0	3·679 ,,	155·0

GROUP III.—Sooted and Silvered Globes in Air.

XIII. June 22, 1906. Sooted globe in air. Barometer pressure = 765·5 millims. Temperature of enclosure = 80° absolute.				XIV. October 9, 1906. Silvered globe highly polished.* Barometer pressure = 758 millims. Temperature of enclosure = 80° absolute.				XV. April 12, 1906. Silvered globe with dull surface.† Barometer pressure = 761·7 millims. Temperature of enclosure = 80° absolute.			
No.	Difference of temperatures. $(t-t')$° C.	Emissivity. e.	Absolute temperature of globe. θ.	No.	Difference of temperatures. $(t-t')$° C.	Emissivity. e.	Absolute temperature of globe. θ.	No.	Difference of temperatures. $(t-t')$° C.	Emissivity. e.	Absolute temperature of globe. θ.
	° C.		°		° C.		°		° C.		°
1	208·6	$3{\cdot}874 \times 10^{-4}$	288·6	1	190·8	$3{\cdot}658 \times 10^{-4}$	270·8	1	204·0	$3{\cdot}671 \times 10^{-4}$	284·0
2	171·4	3·805 ,,	251·4	2	160·4	3·637 ,,	240·4	2	174·0	3·634 ,,	254·0
3	143·6	3·738 ,,	223·6	3	135·9	3·617 ,,	215·9	3	143·6	3·576 ,,	223·6
4	110·4	3·625 ,,	190·4	4	116·5	3·597 ,,	196·5	4	110·0	3·482 ,,	190·0
5	82·0	3·490 ,,	162·0	5	72·0	3·530 ,,	152·0	5	83·4	3·369 ,,	163·4
6	62·7	3·355 ,,	142·7	6	59·5	3·503 ,,	139·5	6	59·7	3·218 ,,	139·7
7	46·6	3·187 ,,	126·6	7	45·8	3·462 ,,	125·8	7	45·3	3·040 ,,	125·3
8	29·3	3·047 ,,	109·3	8	32·8	3·372 ,,	112·8	8	29·6	2·866 ,,	109·6
9	15·2	2·837 ,,	95·2	9	17·0	3·188 ,,	97·0	9	17·6	2·614 ,,	97·6
10	6·1	2·509 ,,	86·1	10	8·2	2·958 ,,	88·2	10	2·2	1·574 ,,	82·2
Dry air let into the enclosure, June 21, 1906.											

$(t-t')$ is the difference of temperatures between globe and its surroundings in degrees Centigrade.

e is the emissivity, or loss of heat, per square centimetre per second per 1° C. of difference of temperature.

$$\text{Stefan's coefficient} = \frac{\text{emission}}{\theta^4-\theta_0^{\,4}} = \frac{\text{emissivity}}{(\theta^2+\theta_0^{\,2})\,(\theta+\theta_0)}.$$

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[*] The enclosure was removed at the end of this experiment, and the silvered globe was found to be just as bright as when hung in position. Globe repolished on October 8, 1906.

[†] When the enclosure was opened at the conclusion of this experiment, the silvered globe had a dull white surface due to condensation of mercury vapour upon the globe.